Kinematics

Vectors and Scalars

- Scalar
  - quantities that only contain magnitude and units.
  - Ex: time, mass, length, temperature
- Vector
  - quantities that contain magnitude and direction (as well as units).
  - Ex: displacement, velocity, force, acceleration

Vector Notation

- We represent a vector quantity by drawing an arrow above the letter.

$$\vec{d}$$

Direction

- The direction of the vector can be described in a number of ways:
  - Common terms (left/right, up/down, forward/backward)
  - Compass directions (north, south, east, west)
  - Number line, using positive and negative signs (+/-)
  - Coordinate system using angles of rotation from the horizontal axis

Adding Vectors

- There are two ways that we will use to add vectors:
  - Scale drawings
  - Algebraically

Scale Drawings

- We draw vectors as lines with an arrow head representing the tip of the vector

- The other end of the vector line is called the tail

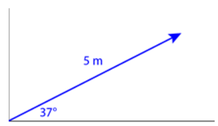
- Choose an appropriate scale
- Draw a line representing the first vector
- Draw a line representing the second vector starting from the tip of the first vector
- Continue until all vectors are drawn
- Join the tail of the first vector to the tip of the last vector
- Measure the length and angle of the joining line

- Example:
  - Add the following vectors: 5 ms^{-1} North and 10 ms^{-1} East

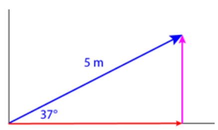
- We could also add these vectors using the Pythagorean theorem, sine laws, and cosine law

Components of Vectors

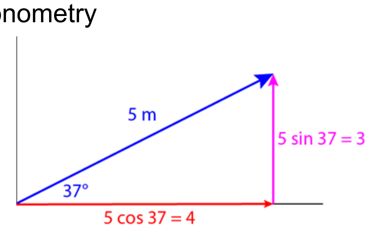
- Consider the vector 5 m 37° N of E



- This vector forms the hypotenuse of a right-angle triangle.



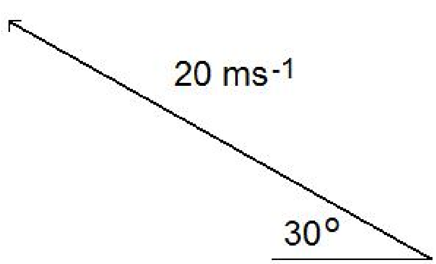
- The sides of this triangle are the **components** of the vector
- These components can be calculated with trigonometry



- This is true for any vector

Example

- 20 ms^{-1} 30° N of W

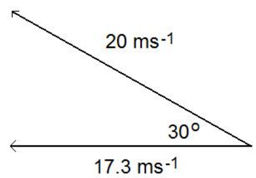


- We can calculate the horizontal side (horizontal component or x component) of the triangle by using cosine.

$$\cos 30^\circ = \frac{x}{20\ ms^{-1}}$$

$$x = (20\ ms^{-1})\cos 30^\circ$$

$$x = 17.3\ ms^{-1}$$

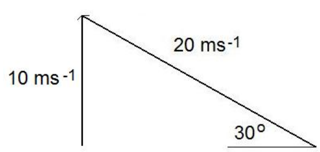


- We can find the vertical side (vertical component or y component) of the triangle by using sine.

$$\sin 30^\circ = \frac{y}{20\ ms^{-1}}$$

$$y = (20\ ms^{-1})\sin 30^\circ$$

$$y = 10\ ms^{-1}$$



Adding with Components

- We can add vectors by using their components
  - Find the components of each vector
  - Add the x components together (remembering direction)
  - Add the y components together (remembering the direction)
  - Put the final vector together (using the Pythagorean therom)

Example

- Add the following vectors:
  - 30 ms^{-1} 25^{o} N of W
  - 50 ms^{-1} 40^{o} S of E

Definitions

- Displacement
  - the change in position of an object. How far the object is away from its starting position. Displacement is a vector quantity.
  - Symbol: s

- Velocity
  - signifies both speed and direction. It is a vector quantity.
  - The change in displacement with respect to time
  - Symbol: v

$$v = \frac{\Delta s}{\Delta t}$$

- Acceleration
  - how rapidly velocity changes. It is a vector quantity.
  - Change in velocity with respect to time
  - It is important to note that acceleration can occur if either speed **or** direction changes.
  - Symbol: a

$$a = \frac{\Delta v}{\Delta t}$$

What is the difference between average and instantaneous?

- Average
  - Measured over a period of time
- Instantaneous
  - Measured over a single infinitesimally small point in time
  - At one exact point in time
  - For example, a speedometer measures instantaneous velocity of a vehicle

Frames of Reference

- Any measurement of position, distance, or speed must be made with respect to a frame of reference.

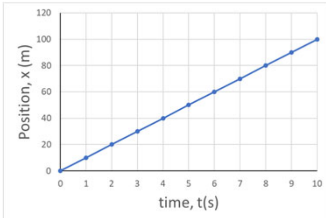
Example

- You are in a car traveling 80 kmh^{-1}. You notice a fly flying towards the front of the car at a speed of 5 kmh^{-1}.
- You are looking at the fly's speed from the reference frame of the car.
- To someone standing on the sidewalk the fly is traveling at a speed of 80 kmh^{-1} + 5 kmh^{-1} = 85 kmh^{-1} with respect to the ground.

- This is why it is always important to know the frame of reference.
- In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame should be specified whenever there might be confusion.
- The term **relative** is used in these cases.

Graphical Representation of Motion

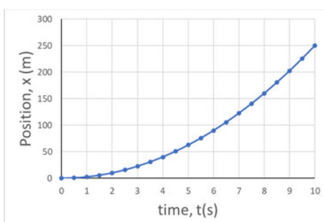
- Consider a car moving with a constant velocity of 10 ms^{-1}.
- The position of the car will be calculated at 1 second intervals and a graph of position vs time will be created for the first 10 seconds of motion.



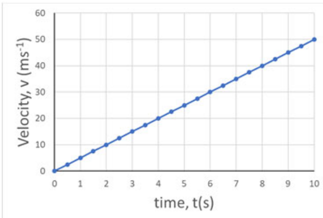
- The slope of the line is constant and is equal to the velocity of the car.

$$\text{slope} = \frac{\Delta x}{\Delta t} = \frac{(100-0)}{(10-0)} = 10\ \text{ms}^{-1} = v$$

- Now consider a car moving with a constant acceleration of 5 ms^{-2}.
- A position vs time graph of this motion is curved.



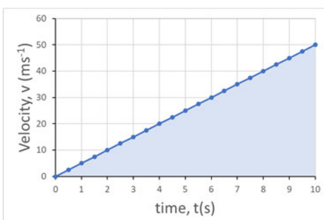
- The slope of this line is not constant.
- The slope increases since the velocity increases.
- A graph of velocity vs time can be obtained by calculating the slope at each point on the position vs time graph.
- This is done by calculating the slope of a tangent line at each point.



- The slope of the line is constant and is equal to the acceleration of the car.

$$\text{slope} = \frac{\Delta v}{\Delta t} = \frac{(50)}{(10)} = 5\ \text{ms}^{-2} = a$$

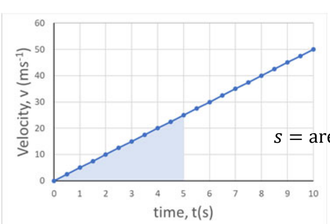
- The area under the curve can also be calculated.



$$\text{area} = \frac{(\Delta v)(\Delta t)}{2} = \frac{(50)(10)}{2} = 250\ \text{m} = s$$

- This is the final displacement of the car.

- The displacement at any time can be calculated by calculating the area under the curve up to that point.
- For example, the displacement at 5 seconds would be



$$s = \text{area} = \frac{(25)(5)}{2} = 62.5\ \text{m}$$

- Similarly, the area under an acceleration vs time graph is velocity.